

# Multistable Feature Binding with Noisy Integrate-and-Fire-Neurons

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**Abstract:** *Homogeneous networks of spiking neurons exhibit synchronization and multistable phase clustering when the interaction is pulselike and delayed. Our simulations show that these phenomena persist in models of more realistic neurons which operate near threshold. This situation implies a high sensitivity to small input fluctuations which appears to be present in the cortex. In this paper we explore the functional consequences of these operation conditions. It turns out that despite the presence of noise coarsely coactivated neurons synchronize and cluster their activity in time. This provides a mechanism for the separation and simultaneous representation of several objects without requiring any structure in the connection weights.*

## 1 Multistable Phase Clustering

In the last years, oscillatory and synchronized neuronal responses have been hypothesized to play a key role in information processing in the brain. Stimulus-dependant synchronous oscillations have been observed in the visual cortex of cats and monkeys [1]. It could be shown that synchronization of neuronal populations may depend systematically on Gestalt properties of the stimulus [2]. Theoretical considerations showed that a time structure of neuronal responses provide a means to code different feature combinations presented in a stimulus (binding-problem) and, at the same time, to avoid the so called superposition catastrophe [3]. Despite much modelling effort, it is still not known how this processing paradigm could be implemented in an efficient algorithm. As part of such a network, neuronal oscillators might provide a powerful tool for this segmentation task, which has e.g. been demonstrated by Sompolinsky et. al. [4].

Additionally, there has been much effort to analyze the dynamics of small populations of neurons in dependence of their coupling strengths, topology, axonal delays and other parameters [5]. One result of these investigations, rather inhibitory than excitatory couplings can be made responsible for stable synchronous activity. [6]. Following this track, we analyzed in previous work synchronization effects in the special case of globally pulse coupled integrate-and-fire neurons. Such models combine an essential biological plausibility with the possibility of rigorous mathematical analysis [7]. We showed that the behaviour of such networks can be studied in terms of a return map, which describes the evolution of the phase difference between two arbitrary integrate-and-fire-oscillators [8]. It turned out that the presence of delays causes drastic changes in the dynamics of the system: With inhibitory couplings and no delays ( $\tau = 0$ ), the neurons desynchronize completely. Delay times  $\tau > 0$  lead -in mathematical terms- to a stable fixed point at zero phase lag, such that a fully synchronized solution can be achieved and is always stable. In contrast, synchronization with excitatory couplings gets weaker with increasing delays, caused by the instability of the respective fixed point at phase

lag zero.

With inhibitory couplings, not only a fully synchronized, stable solution exists, but also other interesting dynamical phenomena can occur. For example, a random initialization leads to multistable clustering. The neuronal population spontaneously breaks into a number of subpopulations, such that each neuron fires synchronously with all the others within its cluster, but the populations themselves fire alternately. The multistable occurrence of an arbitrary, but limited number of synchronous subpopulations (Fig.1) can be understood as a consequence of the coexistence of several stable fixed points within each cluster, which synchronize all neurons within their respective basins of attraction.

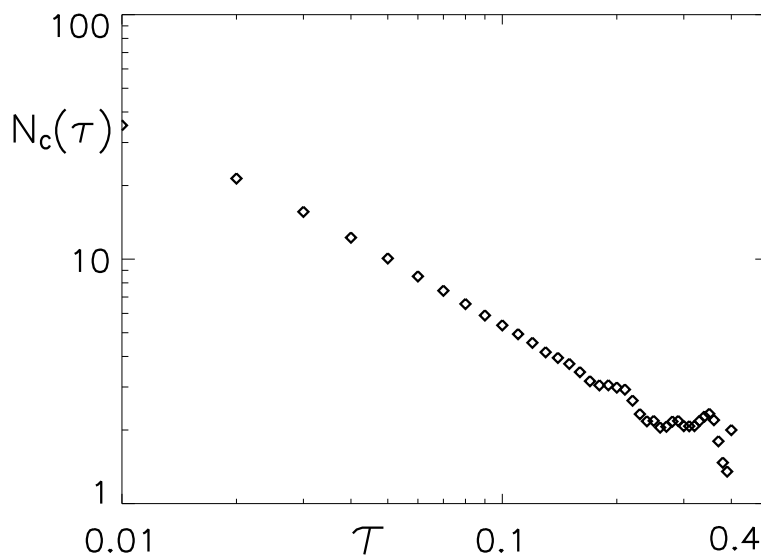


Figure 1: Maximum number  $N_c$  of synchronous clusters, which is equal to the network frequency, in a network of  $N = 100$  pulse coupled neurons, plotted against the delay time  $\tau$ .

The firing frequency of the summed network activity is increased by a factor equal to the number of subpopulations. This phenomenon is analogous to a firing rate increase found in the hippocampus of rats [9]. In the next sections, we discuss the possible functional relevance of the persisting multistability in the more realistic case of stochastic neurons which exploit input fluctuations.

## 2 Application: Segregation and Binding

The basic units of our neural network are  $N$  integrate-and-fire-neurons which obey the following differential equation

$$\begin{aligned}
 dv_i/dt &= -c \cdot v_i + I + I_{int.} + I_{ext.,i} + \eta(t). \\
 o_i &= \begin{cases} 1 & \text{if } v_i > v_{thres} \\ 0 & \text{otherwise} \end{cases} .
 \end{aligned} \tag{1}$$

$v_i$  denotes the membrane potential of neuron  $i$ ,  $c$  the membrane capacity,  $I_{int.}$  the synaptic current caused by the global coupling to the other  $N - 1$  neurons, and  $I_{ext.,i}$  an external current which will be described below. With  $I_{int.} = I_{ext.,i} = 0$ , a constant current  $I \leq c$  raises the membrane potential just below the firing threshold  $v_{thres} := 1$ . Only if the noise  $\eta$  is high enough, the threshold can be crossed and a spike can be emitted. These conditions ensure that a single neuron without additional input fires stochastically with a low firing rate. By emitting a spike, the membrane potential is reset to a refractory value of  $v_{ref} = 0$  and the output variable  $o_i$  of neuron  $i$  is set to 1.

Each neuron is coupled to each other neuron via inhibitory connections. The spikes are transmitted with a delay  $\tau$ ; if neuron  $A$  fires at  $t$ , neuron  $B$  receives this spike at  $t + \tau$ . Thus,  $I_{int.} = \sum_{j=1}^N w_{ij} o_j(t - \tau)$ . The  $w_{ij}$ 's are uniform, i.e.  $w_{ij} := w_0$  for  $i \neq j$  and  $w_{ii} = 0$ .

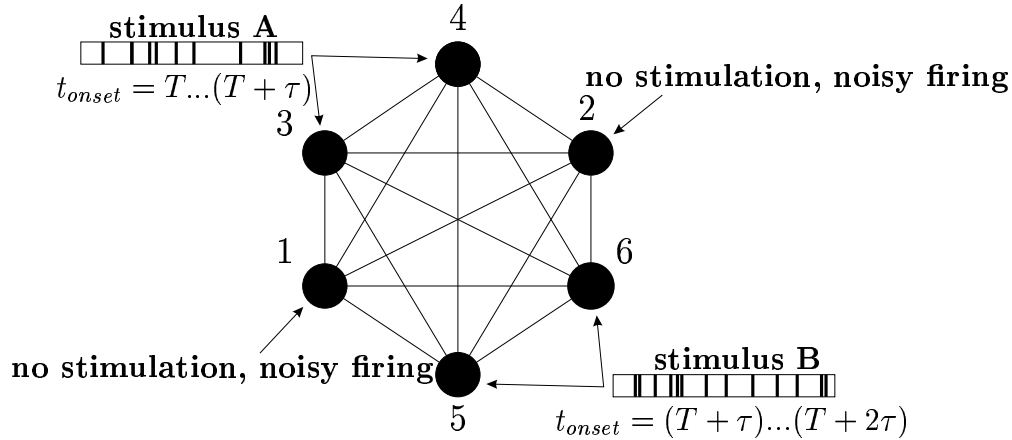


Figure 2: Schematic view on the network: two stimuli, e.g. the summed activity of two pools of external neurons, activate the neurons 3, 4 and 5, 6. Neurons 1 and 2 stay quiescent. Neuron 1 through 6 are coupled globally and inhibitory with delay  $\tau$ .

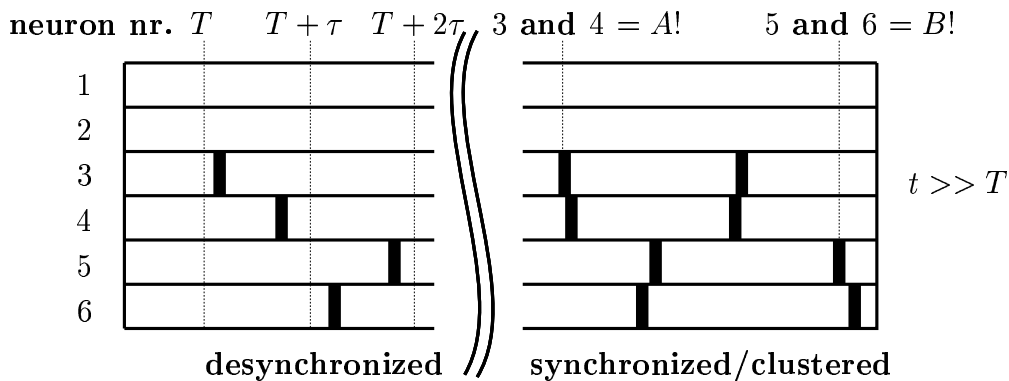


Figure 3: Spiketrains of the six neurons of Fig.2. At stimulus activation time  $t = T$  (left side), the neurons are still desynchronized but activated successively. At time  $t \gg T$ , the neurons belonging to one stimulus have synchronized their activity and two clusters have been formed.

We now suppose the existence of  $M$  stimuli, each of them stimulating a different subgroup of the  $N$  neurons. Furthermore, we assume that the onset of these stimuli does not take place

simultaneously, but successively, as in the following example: Consider a net of 6 neurons and two stimuli  $A$  and  $B$  (Fig.3). Stimulus  $A$  acts on neurons 3 and 4 and is turned on in the time interval  $\Delta t_1 = [T, T + \tau]$ . Stimulus  $B$  acts on neurons 5 and 6 and is turned on in  $\Delta t_2 = [T + \tau, T + 2\tau]$ , while stimulus  $A$  is already acting on neurons 3 and 4. In our simulations, we modeled each stimulus by  $N_{ext}$  neurons which fire each timestep with a probability  $p_{fire}$ . These spikes are summed up and projected onto the main network via the external synaptic current  $I_{ext,i}$  where each spike contributes with an amount of  $I_{sp}$ .

### 3 Results

We simulated a network of  $N = 100$  globally coupled neurons with parameters  $c = 2.5$ ,  $I = 2.5$ ,  $w_{ij} = -0.02$  ( $i \neq j$ ),  $\eta = 0.05$  (Gaussian distribution), and  $I_{sp} = 0.2$ , using a discretized version of Eq.1. Two stimuli  $A$  and  $B$ , each of them projecting onto 18 neurons, were applied. Neurons receiving input from stimulus  $A$  and  $B$  are activated beginning at random time in the intervals  $[T, T + \tau]$  and  $[T + \tau, T + 2\tau]$ , respectively ( $\tau = 10$  timesteps). In Fig.4, 50 spiketrains out of 100 neurons are depicted. Immediately after the onset of the stimuli at  $t = 100$ , the neurons are desynchronized. After a few firing periods, the neurons receiving input from the two stimuli were synchronized in two clusters which fire alternately.

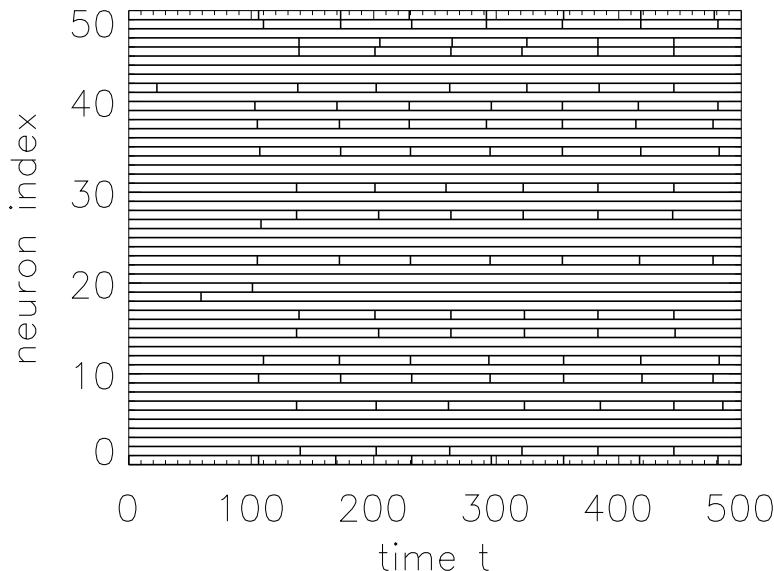


Figure 4: Spiketrains of 50 out of  $N = 100$  neurons over  $\Delta T = 500$  timesteps. The small vertical lines indicate that a neuron emits a spike in the respective time step.

Fig.5 shows the firing patterns of all 100 neurons, a) beginning at time  $t = T$  and b) beginning at time  $t \gg T$ . For the purpose of demonstration, the neurons are arranged in an array of size  $10 * 10$ . This arrangement is completely arbitrary, because of the uniformity of the connection matrix  $w_{ij}$ . During the onset of the stimuli, the activation pattern hardly can be distinguished. After synchronization has taken place, two synchronized groups can clearly be distinguished, while the two stimuli are already simultaneously active. The groups corresponding to stimulus  $A$  and  $B$  appear like a box and a cross, respectively. The temporal segregation is caused both by the successive presentation and the "right" adjustment of the delay time. It uses the property of the multistable clustering of inhibitory coupled neurons to alternately activate two

combinations of features and neurons, which are symbolized by the box and the cross. At the same time, synchronization sharpens this information which is carried by the neuronal pool for further processing. The segmentation is stable up to a noise level of 5-10 percent, depending on the inhibition strength. Additionally, the dynamic does not change significantly by introducing broad distributions of coupling constants and delay times.



Figure 5: Firing patterns of  $N = 100$  neurons a) during onset of stimulation and b) several firing periods later. After synchronization has taken place, it can be distinguished that stimulus  $A$  activated the neurons forming a cross and  $B$  activated the box.

## 4 Summary and Discussion

The phenomenon of multistable clustering offers the following perspectives for information processing with spiking neurons:

- Depending on the delay time, the network splits into a variable number of synchronous subpopulations. On the one hand, this behaviour can be used to bind information by synchronization and on the other hand, it can be used to segregate discrete sets of information "packets" from each other.
- The synchronization of activity which has been spread over a time interval  $\tau$  can be helpful to sharpen information, which has to be transported to further processing stages. This is a mechanism which might be useful to implement synfire chains [11].
- Without adjusting the delay, an arbitrary but limited number of subpopulations can be evoked which corresponds to the number of simultaneously presented stimuli.

Despite these advantages, there still remain some questions, which will be the subject of further work.

- Increasing the noise level, the neurons begin to fire stochastically and do not belong to one specific cluster any more. Fortunately, the effect of clustering and synchronization remains stable.
- The network still cannot handle "overlapping" features, which requires the dynamical binding of one feature or neuronal group to two or more subpopulations in an alternate manner. This might be done by an additional mechanism which controls neurons firing stochastically.

- The relationship of the artificial stimulation paradigm to real biological networks is still not clear. Despite the evidence that real neurons may rest most of the time just below threshold, waiting for activation [10], it should be useful to analyze hippocampal data [9] in order to discover if the mechanism of multistable clustering really plays a role in biology.

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