Adaptation and criticality in human control behaviour

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In many complex systems, extreme events occur more frequently than expected for Gaussian scale-free distributed event magnitudes. Instead, distribution tails are well described by power-laws $p(y) \propto y^{-\gamma}$. Power-law scaling of spatial and temporal correlations has been linked to critical points. Existing models are either simple, but require parameter tuning (intermittency) or non-linear with many degrees of freedom (self-organized criticality) [4].

Critical fluctuations are also found in stock market log-returns and human motor control errors during upright standing or stick balancing [1]. These extreme fluctuations might appear suboptimal and for returns are suspected to reflect inefficiencies. We demonstrate, that simple control systems can self-tune to critical points as a result of highly efficient control. This mechanism is further shown to reproduce human balancing behaviour in great detail.

Stabilisation annihilates information about future dynamics of a system. For example, it is only possible to predict where a stick is going to fall once it starts falling. The more efficient the control, the less can be learned about its dynamics by observation.

More generally, consider a system close to a fixed point with expected dynamics in the distance from some target value $y(t) \sim y^*(t)$. The expected amount of information about $y$ (Fisher information) $\gamma = \frac{\partial^2}{\partial y^2} \ln p(y)$ that a noisy observation of $y \sim \mathcal{N}(y^*, \sigma)$ and $\hat{y} = \mathcal{N}(y^*, \sigma^2)$ contains vanishes close to the origin where $\gamma \rightarrow 0$. Since $1/\gamma$ is a lower bound for the mean squared error of an unbiased estimator for, stabilisation (minimizing $\gamma$) resolves the system towards a critical point.

A minimal model
Consider a discrete-time random map:

$$y_{n+1} = \lambda y_n + \epsilon_n$$

with Gaussian distributed random variables $\epsilon$. Control by removing an estimation of $y_{n+1}$ in the next timestep:

$$y_{n+1} = \lambda (y_n - \hat{y}_n) + \epsilon_n$$

The parameter $\lambda$ is estimated optimally from the last two observations $[\hat{y}_n, \hat{y}_{n-1}]$

$$\hat{y}_{n+1} = \frac{y_n - \hat{y}_n}{1 - \lambda}$$

Eqs. (2) and (3) generate power-law distributed fluctuations with pdf exponent $\beta = 2$, independent of $\lambda$ and $\beta$ [3].

The systems dynamics is described by an SDE

$$\frac{dy}{dt} = -\lambda (y - y^*) + \eta(t)$$

with “white noise” $\eta \sim \mathcal{N}(0,1)$ and an estimation of $y(t)$ at $t = 0$

$$y(t) = y^* \left( 1 - e^{-\lambda t} \right) + \int_0^t \lambda e^{-\lambda \tau} \eta(t) d\tau$$

The parameter $1/\lambda$ is estimated using a maximum likelihood estimator with an exponential decaying window. It can be written differentially as

$$\frac{d\hat{y}}{dt} = -\lambda (\hat{y} - y^*) + \int_0^t \hat{y} \left( 1 - e^{-\lambda (t - \tau)} \right) d\tau$$

Do power-laws indicate inefficiency?
Surprisingly, fast adaptation minimizes mean balancing errors by tolerating rare, large errors in favor of the removal of random trends. Corresponding pdf scaling exponents lie in the highest range of those observed experimentally.

Summary
• Human behavior shows power-law distributed fluctuations.
• Control using optimal short-time estimation of parameters can induce critical points as attractors.
• A model with realistic constraints on reaction time and control strength can reproduce many features of human control behavior.
• Minimal mean squared control errors coincide error distributions in between the Lévy- and the Gaussian regime.

Realistic balancing model*
Control of an unstable system is modelled by removing a term proportional to a prediction based on a linear trend in the most recent observations prior to a fixed reaction time. The controller has three free parameters:

• reaction time $t_r$
• gain factor $\gamma \sim 1$ – effective time constant to bring to zero and
• time constant for “forgetting old observations” $\tau_c$

Good fits are obtained for cautious control ($\gamma < 1$) and fast adaptation ($\tau_c \ll t_r$).

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“Volatility clusters”

Outlook
• Measure subjects’ control parameters
• Do different control tasks or objective functions cause different error distributions after sufficient training?
• Learning on slower timescales
• Microscopic systems (e.g. a network)
• Minimising local information as a general principle for self-organised criticality

References